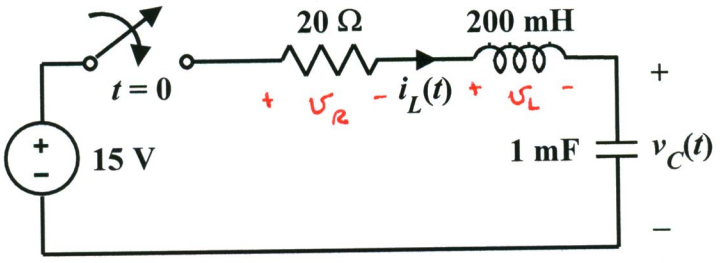
EE 2240

Problem #04

Find $v_C(t)$ for $t>0$ if $v_C(0)=0$ V and $i_L(0)=0$.



$i_L(t) = (1\text{mF})\,\dot{v}_c \Rightarrow \dot{v}_c = 1000\, i_L \Rightarrow \dot{v}_c(0) = 1000\, i_L(0) = 0$

$v_R = (20\Omega)\, i_L = 0.02\, \dot{v}_c$

$v_L = (200\text{mH})\,\frac{di_L}{dt} = 0.0002\, \ddot{v}_c$

$v_L + v_R + v_c = 0 \Rightarrow 0.0002\,\ddot{v}_c + 0.02\,\dot{v}_c + v_c = 0$

$\therefore \ddot{v}_c + 100\,\dot{v}_c + 5000\, v_c = 0$

$r^2 + 100r + 5000 = (r+50)^2 + (50)^2$

$\Rightarrow r = -50 \pm j50$

$v_c(t) = e^{-50t}(K_1 \cos 50t + K_2 \sin 50t) + K_3$

$v_c(\infty) = 15\text{V} \Rightarrow K_3 = 15$

$\therefore v_c(t) = e^{-50t}(K_1 \cos 50t + K_2 \sin 50t) + 15$

$\dot{v}_c(t) = -50\, e^{-50t}(K_1 \cos 50t + K_2 \sin 50t)$
$+ e^{-50t}(-50 K_1 \sin 50t + 50 K_2 \cos 50t)$

$v_c(0) = K_1 + 15 = 0 \Rightarrow K_1 = -15$

$\dot{v}_c(0) = -50K_1 + 50K_2 = 0 \Rightarrow K_2 = K_1 = -15$

$\therefore v_c(t) = e^{-50t}(-15\cos 50t - 15 \sin 50t) + 15$

$= 15\left[1 - e^{-50t}(\cos 50t + \sin 50t)\right] \quad V,\ t \geq 0$